M.A.(Managerial Economics) Seminar

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Thermodynamics Formulation of Demand-Side Economics

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MAHIDOL UNIVERSITY NAKHONSAWAN CAMPUS PROJECT

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ศูนย์ฟิสิกส์ทฤษฎีและปรัชญาธรรมชาตินครสวรรค์
"อาศรมวิชชาลัยเพื่อการค้นคว้าขั้นก้าวหน้า"
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Physics approaches to Economy/Economics

Physics approaches to Economy/Economics

1. Tools of Statistical Mechanics and its Path Integral technique (Time Series Data, Panel Data)

- SM of Agent Base Model,
- SM of Complex Networks, Path Integral Technique

2. Substantive Approach

"economic activities are viewed as thermodynamic process"

3. Formal analogy “*mathematical analogies*” – in Finance

- Quantum Finance (Quantum Mechanics, Quantum Field Theory in Finance, Path Integrals Quantum Mechanics in Finance)
- Financial Mathematics as Fluid Dynamics (Black-Scholes-Merton Model)

4. Formal analogy “*mathematical analogies*” with “*empirical postulates of nature of aggregated complex system*” - Thermodynamics Formulation of Economics

physical | social

• System’s Name (Phys)	System’s Name (Social)
• System’s Properties (Phys)	System’s Elasticity (Social)
• System’s State (Phys)	System’s State (Social)
• Influence (Phys)	Influence (Social)
• Laws telling properties to change its state (Phys)	Laws telling elasticity to change its state (Social)
• Agent’s function to obey laws (Phys)	Person’s social duty to do (Social)
• Systematic Uncertainty (Phys)	Systematic Uncertainty - Risk (Social)
•	Agent’s Choice (Option) Preference, Memory (Social)

Physics and Economics

Physics: [how things change]

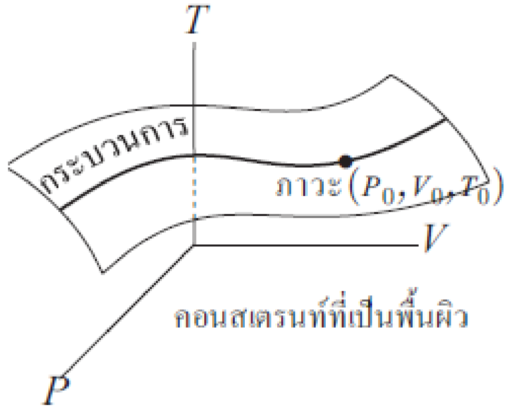
- Manifold as home of laws, e.g EoS in Thermodynamics
- Postulate laws and Empirical laws with prediction power
- Toward unification picture
- Short Relaxation Time
- Controlled conditions
- Coordinates and Potential (or Field)
- Richer in fundamental units (6)
  (meter | second | kg | Coulomb | Kelvin | mole)

Traditional Economics: [making choices to allocate limited resources]

- No EoS (so far)
- Postulates with Empirical formulae with prediction power
- Unification incomplete: separated by “schools of thoughts”
- Long Relaxation Time/hardly attaining equilibrium state
- Frequently minor disturbed or big shock
- Potential (or Field) and Coordinates
  (different concept of coordinates in that of EoM or EoS)
- Fewer in fundamental units (> 4)
  (money unit | time unit | # of Goods | # of Agents (Consumer, Seller, Company)) (more units as more types of agents or goods)

- incomplete meaning of ideal gas laws (no description of cause and effect)

$$V \propto T \qquad V \propto 1/P \qquad P \propto T$$



- Three empirical facts form EoS relating 3 coordinates (2 d.o.f.)

  (ideal gas)

$$PV = nRT$$

in general hydrostatics system $\qquad g(V, P, T) = 0$

Empirical axioms: *V, P* and *T* are “coordinates of thermodynamics space” (*entropy* fixed)

- Thermodynamics Potential "field" $U = U(P, V, T)$

Thermodynamics Potentials | Coordinate Space Variables | Energy Transfer

- In economics: "stock quantity" or "flow quantity".

Flow: work and heat
Stock: **potential or coordinates?**

Previous Speculations on Similarities (Thermo vs Econ)

1. Two dual states: at Equilibrium
2. Maximization of one quantities (Preference)-Minimization of the other
3. Conservation Constraint

[e.g. Fisher (1892) Walras (1909) Lisman (1949) Saslow (1999) and Smith and Foley 2008]

ADDITIONAL SPECULATIONS ON SIMILARITIES (THERMO VS ECON)

1. Aggregated Nature
2. Truly Endogenous Function
3. Effect Structure Diagram
4. A scalar derived from conservative variables must be a temperature
5. Unmeasurable nature of *U*-Thermo, Wealth-Econ (Minimize-demand size) |
   S –Thermo, Economic Entropy (Maximize-demand size)

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Previous Literatures

- Fisher (1892) – Thermodynamics analogy to grad. of Utility w.r.t. quant. of goods

: grad. of Internal Energy (mismatching!)

- Walras (1909) – force of potential energy analogy to grad. of Utility w.r.t. quant. of goods
- Abandoned for many years until Samuelson's critique (1960)

"The formal mathematical analogy between classical thermodynamics and mathematic economic systems has now been explored. This does not warrant the commonly met attempt to find more exact analogies of physical magnitudes - such as entropy or energy - in the economic realm. Why should there be laws like the first or second laws of thermodynamics holding in the economic realm? Why should 'utility' be literally identified with entropy, energy, or anything else? Why should a failure to make such a successful identification lead anyone to overlook or deny the mathematical isomorphism that does exist between minimum systems that arise in different disciplines".

(not yet done really! – need of EF. St. concept)

- No concept of EoS in Econ., and as a result, no concept of potential in Econ

  (Debrue 1972, Mas-Colell 1985)

  "Economics, because it does not recognize an equation of state or define prices intrinsically in terms of equilibrium, lacks the close relation between measurement and theory physical thermodynamics enjoys."

  (Smith and Foley 2008)

- Similarities of Thermodynamics and Economics:

$$g(Q, Pr, \mathrm{MRS}) = 0.$$

Smith and Foley 2008 proposed EoS.
(Utility as Entropy: Two Types of Good)

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Extensive Co., Intensive Co., First Law

mechanical pair

extensive coordinate from mechanical definition, (X) — size-dependent

intensive coordinate from mechanical definition, (Y) — size-independent

0 for closed system

$$dU = \delta W + \delta Q + \cancel{\mu dn}$$

1st Law (conservation)

heat term $\delta Q = T dS$

work term, $\delta W = Y dX$

$$\left.\begin{matrix} X \\ T \\ Y \\ S \end{matrix}\right\} \text{coordinates}$$

internal energy $U = U(S, X_i, n)$. — Field or Potential

EoS $g_i(X_i, Y_i, T) = 0$ — constraint

Truly Endogenous Function

Consider $V = V(P,T)$ (seems fine) however in fact it is

A mapping with non-composite function from "cause" variable to $V = V \circ P(T) = V(P(T))$

Hence truly endogenous function are, for examples,

$$V = \tilde{V}(P)$$
$$P = \tilde{P}(T)$$
$$T = \tilde{T}(P)$$

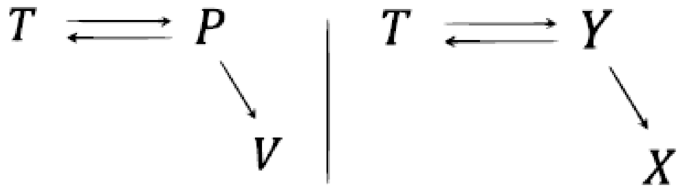
In which argument of function in the cause and the value of function is the effect.

This is performed with directed graph.

Diagram of Effect Structure: Class I

<u>Class I</u>: (two opposite-direction arrows linking T and Y)

$$\text{Class I:} \qquad V = \frac{nRT}{P} \qquad \Longrightarrow \qquad X = k\left(\frac{T}{Y}\right)$$



Class I

Figure 1. Hydrostatics system is an example of effect structure in Class I type EoS. Temperature T can affect pressure P and so reversely. P can affect volume V. However V can not affect any other variables. On the right, for generalization, X and Y are written in the diagram.

Arrows represent truly endogenous function and they represent direction of cause and effect.
Initial change in T or Y are possible from both exogenous effect and endogenous effect.

Diagram of Effect Structure: Class II

Class II: (two opposite-direction arrows linking T and X)

$$\text{Class II:} \qquad \mathcal{M} = C\frac{B_0}{T} \qquad \Longrightarrow \qquad X = k\left(\frac{Y}{T}\right)$$

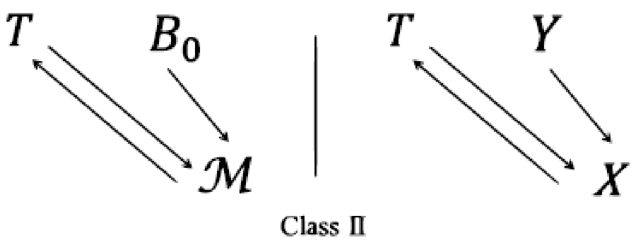


Figure 2. Paramagnetic system is an example of effect structure in Class II type EoS. Temperature T can affect magnetization $\mathcal{M}$ and so reversely. External magnetic field intensity B_0 can affect $\mathcal{M}$. However B_0 can not be affected by other variables.

Effect Structure Rules
– A Proposal of Formal Procedure

Aspects noticed by our study and shall be applied as criteria of the status of an EoS, $g(X, Y, T) = 0$ of a system. These come as some novel rules for an empirical relation to be considered as an EoS. These rules are

1. Number of the arrows is three.
2. There is at least one arrow pointing $Y \to X$.
3. Only T and(or) Y (apart from causing truly endogenous effects) can also be considered as exogenous influence or a shock[1].

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3-Dim Space of Demand-size Thermo. Economics

Observation empirical under some assumption (just as when considering the "ideal" gas case), a market is "perfect competitive" with "information symmetric" at equilibrium.

Proposing "-Price" as intensive coordinate $-Pr$ with unit $\dfrac{\text{Money Unit}}{\text{Unit Quantity of Good}}$

Proposing "Demand Quantity" as extensive coordinate Q^{d} with unit Quantity of Good

Proposing "Average personal Wealth" as temperature $\varphi = \dfrac{\mathcal{W}}{N}$ with unit $\dfrac{\text{Money Unit}}{\text{Unit Quantity of Consumer}}$

$T = 2U/(3Nk_{\mathrm{B}}) = 2U/(3nR)$

The EoS $g(Q^{\mathrm{d}}, Pr, \varphi) = 0$

Total Wealth Function

- Total Wealth function $\mathscr{W} = \text{wealth in happiness} + \text{wealth in asset}$

$$\mathscr{W} = \mathscr{W}(Q^{\mathrm{d}}, Pr, \varphi)$$

is a potential (field) function of $(Q^{\mathrm{d}}, Pr, \varphi)$. It plays a role of internal energy.

Generalized Utility function (heat term)

- Generalized Utility function (heat term) $\delta \mathscr{Q}^{\mathrm{d}}_{\mathrm{util}} = \varphi \, \mathrm{d}S^{\mathrm{d}}$

$$\delta \mathscr{Q}^{\mathrm{d}}_{\mathrm{util}} = \delta(\text{pleasure or opportunity of ownership in assets}) + \delta(\text{happiness of utilizing commodity})$$

unlike microeconomics, maximizing of $\delta \mathscr{Q}^{\mathrm{d}}_{\mathrm{util}}$ is not derived by optimizing wr.t. Q^{d}_{j} (coordinates) under a budget constraint. Instead generalized utility is maximized with 2nd law. This is achieved even for one commodity consumption from expenditure or even without expenditure $\delta W^{\mathrm{d}} = -Pr\, \mathrm{d}Q^{\mathrm{d}}$, but only

With $\mathrm{d}S^{\mathrm{d}} \neq 0$. The change $\Delta S^{\mathrm{d}} = \int \delta \mathscr{Q}^{\mathrm{d}}_{\mathrm{util}} / \varphi$ is change in generalized utility per unit of average personal wealth.

Carathéodory's axioms

- "thermal" states

$$f(Pr_1, Q_1^{\rm d}) = \varphi_1 \qquad \text{and} \qquad f(Pr_2, Q_2^{\rm d}) = \varphi_2$$

and so on…

- Expenditure (work term) of demand-side $\delta W^{\rm d} = -Pr\,{\rm d}Q^{\rm d}$
- Economics version of adiabatic process (heat = 0),

Idealistically this is a spending without gaining any utility.
This is a strictly 1st law according to Carathéodory's axioms (isentropic process).

$$\mathrm{d}\mathscr{W} = \delta W^d = -Pr\,\mathrm{d}Q^{\rm d}$$

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Carathéodory’s axioms

- “entropic” states

$$s(Pr_1, Q^{\mathrm{d}}_1) = S^{\mathrm{d}}_1 \quad \text{and} \quad s(Pr_2, Q^{\mathrm{d}}_2) = S^{\mathrm{d}}_2$$

and so on…

heat term (demand-side) heat term $\delta \mathscr{Q}^{\mathrm{d}}_{\mathrm{util}} = \varphi \, \mathrm{d}S^{\mathrm{d}}$

(reversible) is the generalize utility (heat) gained from spending.

- For an economics version of isovolumic process (work = 0),

$$\mathrm{d}\mathscr{W} = \delta \mathscr{Q}^{d}_{\mathrm{util}} = \varphi \, \mathrm{d}S^{\mathrm{d}}$$

Idealisticly this is a gaining of utility without spending.

EFFECT STRUCTURE: CLASS III

- Why are we assured of the space $(Q^{\mathrm{d}}, Pr, \varphi)$?

  Following "The Effect Structure Proposal" we may write and sketch

$$Q^{\mathrm{d}} = kf(Pr, \varphi) \quad \text{or for generalization,} \quad X = kf(Y, T)$$

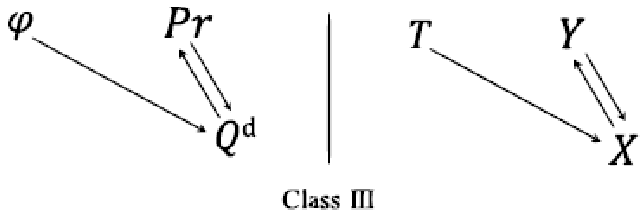
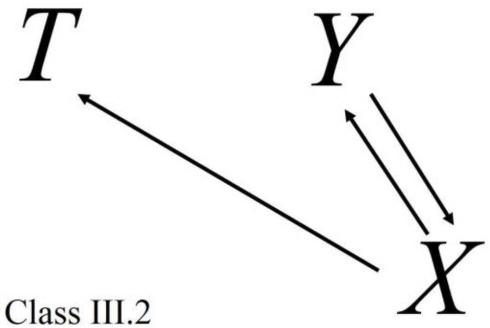


Figure 3. Effect structure diagram of a group of consumers in perfect-competitive market. It is considered as Class III type EoS. Personal wealth φ can affect demand quantity Q^{d}. Price Pr can affect Q^{d} and so reversely. φ can only affect Q^{d}.

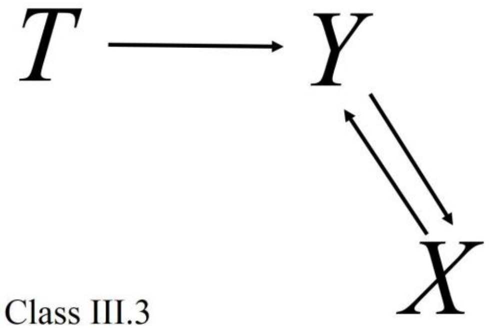
- This obeys the "Criteria Rules" suggesting a Class III
  Class III: (two opposite-direction arrows linking *Y* and *X*)

Above is Class 3.1.
Class 3.2 ($X \rightarrow T$) | Class 3.3 ($T \rightarrow Y$) | Class 3.4 ($Y \rightarrow T$) are also allowed:

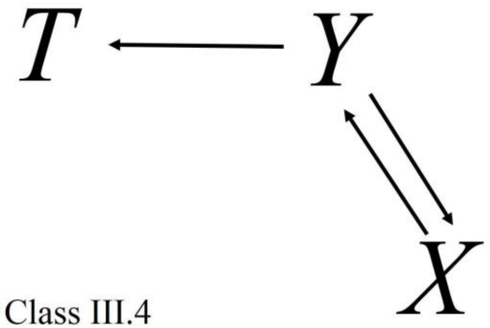
Class 3.2 | Class 3.3 | Class 3.4



Class III.2



Class III.3



Class III.4

Thermal Contact (wealth contact)

Two consumers with different personal wealth: φ_1, φ_2

Can share by: marriage, partners, adoption, family or being together as a household

At equilibrium, each has equal personal wealth (under some assumptions).

What is to be transferred?

Answer: Generalized Utility (assets ownership + happiness) all in money unit. Assets ownership can be transferred with or without expenditure. For thermal contact, it is without expenditure (work term).

Forms of Gen. Utility differ for types of commodity or assets and the way one consumes, e.g. for food, utility is likely to be happiness of eating;
for furniture, utility is ownership and happiness of using them;
for money, utility is ownership and pleasure of opportunity of investment or spending.

Isothermal process

Group of consumers with one equally constant average personal wealth: φ

Processes are governed by Smith's price mechanism: law of demand

(Smith's microeconomics is a specific case of our more generalized theory)

Isobaric Process

φ

One single type of commodity: with one constant price

Demand quantity Q^{d} depends on how rich (in average) φ the consumers are.
Richer people can buy for eating or using more.

0^{th} Law

Existing of average personal wealth φ with personal wealth equilibrium (slowly approaching)

1^{ST} LAW

Existing of wealth function $\mathscr{W} = \mathscr{W}(Q^{\mathrm{d}}, Pr, \varphi)$ with conservation:

$$\mathrm{d}\mathscr{W} = \delta \mathscr{Q}^{\mathrm{d}}_{\mathrm{util}} + \delta W^{\mathrm{d}}$$

2^{ND} LAW

Existing of entropic function S^{d} interpreted as generalized utility/(average personal wealth)

$$\Delta S^{\mathrm{d}} \geq \int \left(\delta \mathcal{Q}^{\mathrm{d}}_{\mathrm{util}} / \varphi\right)$$

In any processes, Gen. Utility/(average personal wealth) does not decrease.

Understanding meaning of S^{d} as Gen. Utility/(average personal wealth):

"Rich people (higher personal wealth) have less happiness in using product.
They tend to look over the value of things,
because it is easy for them to buy the products (service) again whenever they want.
This is opposite to the poor."

3^{RD} LAW

At zero “average personal wealth” (temperature) state,

Gen. Utility/(average personal wealth) (entropy) is zero.

It takes infinite steps to reach the zero average personal wealth state (zero temperature).

“Wealth includes happiness, there is no way to take it away completely.”

“If zero average personal wealth state exists (no assets and no even “passion” of happiness), there is zero utility for him/her. (no happiness of utilizing, no ownership)”

(The zero average personal wealth state is analogy **nirvana state in Buddhism Philosophy)**

Modelling of EoS: Demand Function

- Coordinates $(Q^{\mathrm{d}}, Pr, \varphi)$
- Potential or Field $\mathscr{W} = \mathscr{W}(Q^{\mathrm{d}}, Pr, \varphi)$ with natural coordinates (from first law) S^{d} and Q^{d}
  (with EoS, $g(Q^{\mathrm{d}}, Pr, \varphi) = 0$ hence independent coordinates reduce to 2 d.o.f.
- Truly Endogenous Functions

$$Q^{\mathrm{d}} = \tilde{Q}^{\mathrm{d}}(Pr) \qquad Pr = Pr(\tilde{Q}^{\mathrm{d}})$$

$$Q^{\mathrm{d}} = \tilde{Q}^{\mathrm{d}}(\varphi)$$

- No truly endogenous function between Pr and φ
- Effect structure is described by diagram (directed graph)
- Each arrow must be Truly Endogenous Function

$$\begin{tikzcd} \varphi \arrow[drr] & & Pr \arrow[d, shift left] \\ & & Q^{\mathrm{d}} \arrow[u, shift left] \end{tikzcd}$$

Modelling of Demand Function EOS

$$q(Q^{\mathrm{d}}, Pr, T) = 0$$

$$\mathrm{d}Q^{\mathrm{d}} = \left(\frac{\partial Q^{\mathrm{d}}}{\partial \varphi}\right)_{Pr} \mathrm{d}\varphi + \left(\frac{\partial Q^{\mathrm{d}}}{\partial Pr}\right)_{\varphi} \mathrm{d}Pr$$

$$\begin{aligned} \beta_{Pr} &\equiv \frac{1}{Q_0^{\mathrm{d}}}\left(\frac{\partial Q^{\mathrm{d}}}{\partial \varphi}\right)_{Pr} = \frac{E_{\varphi}^{\mathrm{d}}}{\varphi_0} \\ \kappa_{\varphi} &\equiv -\frac{1}{Q_0^{\mathrm{d}}}\left(\frac{\partial Q^{\mathrm{d}}}{\partial Pr}\right)_{\varphi} = -\frac{E_{Pr}^{\mathrm{d}}}{Pr_0} \end{aligned}$$

as in microeconomics

$$Q^{\mathrm{d}}(Pr, \varphi) = Q_0^{\mathrm{d}}\left[1 + \beta_{Pr}(\varphi - \varphi_0) - \kappa_{\varphi}(Pr - Pr_0)\right]$$

EoS

EoS in form of regression equation

$$Y = 1 + \beta_{Pr} X_1 - \kappa_{\varphi} X_2 + u$$

u is an error term.

$$\begin{aligned} Y &\equiv \frac{Q^{\mathrm{d}}}{Q_0^{\mathrm{d}}} \\ X_1 &\equiv \varphi - \varphi_0 \\ X_2 &\equiv Pr - Pr_0 \end{aligned}$$

One can use econometric methods for
e.g. cross-section or time series data with the EoS

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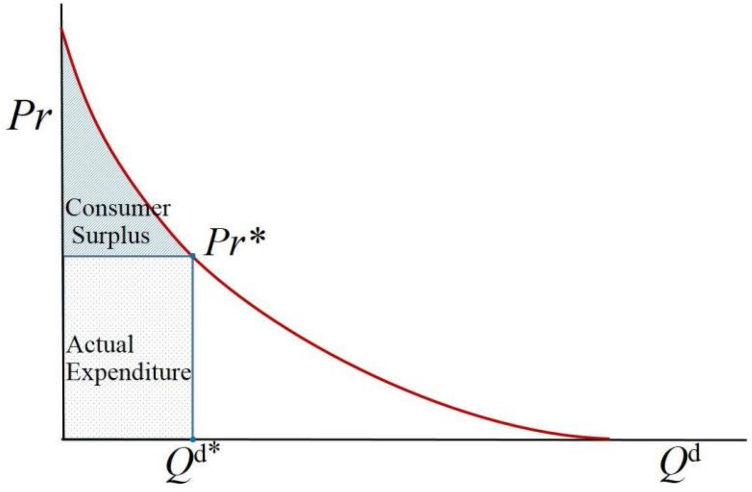
CONSUMER SURPLUS

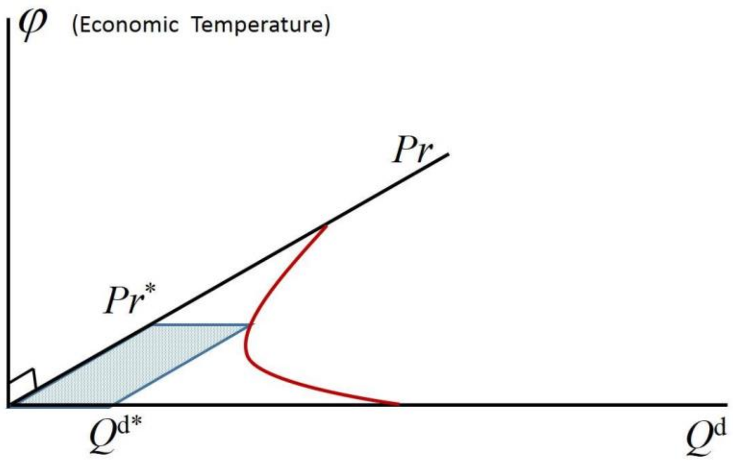
Consumer surplus in economics is without realization of φ coordinate.

It is the further utility gained from the utility at value of actual expenditure (at equilibrium price).



In 3 coordinates, consumer surplus does not fit into our theoretical foundation. We argue that the surplus area (work area) should not be interpreted as utility which is in fact heat term.

Utility should be the product of Economic Temperature φ and Economic Entropy S^d .
(as *T*-*S* diagram)



When spending, we gain utility, therefore why not utility (happiness) should be considered inclusively in the conservation law (first law). Most people think only asset is conservative (but in fact not only that)

Happiness is real in the conservation Eq.

When spending, we gain utility, therefore why not utility (happiness) should be considered inclusively in the conservation law (first law). Most people think only asset is for conservation (which I disagree.).

Economics agents are in existence not because of only asset or money.

For them to spend or to live for something, they need to have memory of happiness or passion.

Hence,

$$\text{total wealth} = \text{asset} + \text{happiness}$$

Helmholtz Free Energy

Consider

$$\mathcal{F} = U - TS = -\cancel{PV}$$

$$\mathscr{F} = \mathscr{W} - \varphi S^{\mathrm{d}} = -\cancel{Pr\,Q^{\mathrm{d}}}$$

$$\mathrm{d}\mathcal{F} \le Y\mathrm{d}X - S\mathrm{d}T$$

$$\mathrm{d}\mathscr{F} \le -Pr\,\mathrm{d}Q^{\mathrm{d}} - S^{\mathrm{d}}\mathrm{d}\varphi$$

Hence at Pr^*, Helmholtz Free Energy ~~= total expenditure~~

.

"Consumers buy goods because the purchase makes them better off"

p.132 sec. on Consumer Surplus, Microeconomics 8th Ed. Pindyck and Rubinfeld

$$\mathrm{d}\mathscr{F} \leq -Pr\,\mathrm{d}Q^{\mathrm{d}} - S^{\mathrm{d}}\mathrm{d}\varphi$$

$$\mathrm{d}\varphi < 0 \quad \longrightarrow \quad -S^{\mathrm{d}}\mathrm{d}\varphi > 0$$

they gain more than what they pay!

This time, left with thermal term (Gen. Util) term, $-S^{\mathrm{d}}\mathrm{d}\varphi > 0$

As, this is really left with a "better off".

Further Works

This is the beginning step to

- Applying Legendre transformation to find other potential and find economics meaning
- Using the EoS with other existing literature in the field of statistical mechanics in finance in which partition function and Hamiltonian are without solid ground of demand-side economics mechanical pair (price and demand quantity) found here with help of effect structure. This gives contribution to financial physics modelling
- Trying with real data | confronting with traditional economics prediction
- Concepts of I.C. curve and multiple EoS
- Heat engine
- Supply-side