NAS, Centre for Theoretical Physics and Natural Philosophy

Mahidol University, Nakhonsawan Campus

Associate Degree Program Computer Science

# Quantum Mechanics

Assignment-I, Due-Date: 5th February , 2024

Timing: Before 5pm | Sciama's Terma 2024, II-Semester | Max mark: 50

**Attempt any five questions totalling 50 Marks**

1. (a) Construct an orthonormal basis starting with $\vec{A} = 3\hat{\imath} + 4\hat{\jmath}$ and $\vec{B} = 2\hat{\imath} - 6\hat{\jmath}$. **(5)**

   (b) Can you generate another orthonormal basis starting with these basis? If so, produce another. **(5)**

2. (a) Show how to go from the basis

$$|I\rangle = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}, \ |II\rangle = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}, \ |III\rangle = \begin{pmatrix} 0 \\ 2 \\ 5 \end{pmatrix},$$

   to the orthonormal basis:

$$|1\rangle = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}, \ |2\rangle = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}, \ |3\rangle = \begin{pmatrix} 0 \\ 2 \\ 5 \end{pmatrix},$$

   **(10)**

3. (a) Prove the Schwarz inequality. **(5)**

   (b) When will this equality be satisfied? Does this agree with your experience with arrows? **(2)**

   (c) Prove the Triangle inequality starting with $|V+W|^2$. You must use $Re\langle V|W\rangle \leq |\langle V|W\rangle|$. Find the condition for equality. **(3)**

4. In a vector space $\mathbb{V}^n$, prove that the set of vectors $\{|V_\perp^1, V_\perp^2, V_\perp^3, ...\rangle\}$ orthogonal to any vector $|V\rangle \neq 0$, form a vector space $\mathbb{V}^{n-1}$. **(10)**

5. Suppose $\mathbb{V}_1^{n_1}$ and $\mathbb{V}_2^{n_2}$ are two subspaces such that any element of $\mathbb{V}_1$ is orthogonal to any element of $\mathbb{V}_2$. Show that the dimensionality of $\mathbb{V}_1 \oplus \mathbb{V}_2$ is $n_1 + n2$. **(10)**

6. An operator $\Omega$ is given by the matrix

$$\Omega = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \tag{1}$$

Find its action. **(10)**

## Best wishes