NAS, Centre for Theoretical Physics and Natural Philosophy

Mahidol University, Nakhonsawan Campus

NWTP-512

Quantum Mechanics

Assignment-II, Due-Date: February 29, 2024

Timing: Before 9 AM | Sciama's Terma 2024, II-Semester | Max mark: 40

Attempt All

1. (a) Show that a product of unitary operators is unitary. **(5)**

   (b) Assuming on the backdrop 1) What a determinant is, 2) that $det\Omega^T = det\Omega$ (where, T is Transpose), 3) That the determinant of the product of matrices is the product of the determinant [If you do not know, verify these properties for a two dimensional case

$$\Omega = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$$

   with $det\Omega = (\alpha\delta - \beta\gamma)$]. Prove that the determinant of a unitary matrix is a complex number with unit modulous. **(5)**

2. Verify that the following matrices are unitary.

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}, \ \frac{1}{2}\begin{pmatrix} 1+i & 1-i \\ 1-i & 1+i \end{pmatrix}.$$

   Verify that the determinant is of the form $e^{i\theta}$ in each case. Are any of the above matrices Hermitian? **(5)**

3. The trace of a matrix is equal to the sum of its diagonal matrix elements

$$trace\Omega = \Sigma_i \Omega_{ii}$$

   Show that

   (a) $Tr(\Omega\Lambda) = Tr(\Lambda\Omega)$ **(5)**

   (b) $Tr(\Sigma\Lambda\theta) = Tr(\Lambda\theta\Omega) = Tr(\theta\Omega\Lambda)$ (Cyclic permutation) **(5)**

   (c) The trace of an operator is unaffeted by the unitary change of a basis $i \to Ui$. **(5)**

4. Show that the determinant of a matrix is unaffected by unitary change of basis. **(5)**

5. (a) Find the Eigen values and normalized eigen vectors of the matrix **(5)**

$$\Omega = \begin{pmatrix} 1 & 3 & 1 \\ 0 & 2 & 0 \\ 0 & 1 & 4 \end{pmatrix}$$

(b) Is the matrix Hermitian? Are the Eigen vectors orthogonal?

Best wishes