



Assignment 2
Ph. D. Coursework, NAS-MUNA
Symmetries & Lie Algebra in Physics
(NWTP 702)
Instructor: Kumar Abhinav
Date: October 13, 2023

Due on October 26, 2023

Semester 1

Total marks: 42

Instructions

- I. Submit answers both in hard and soft (through e-mail) copies. Do not waste time by typing it out.
- II. Use either blue or black ink.
- III. Delay in submission may reduce marks.
- IV. Individual marks are given in parentheses.
- V. Attempt questions 1, 5, and 6, but **any two** from questions 2, 3, and 4.

Questions

1. Consider the group elements,

$$g_1 = (12)(34) \text{ and } g_2 = (14)(32),$$

of the permutation group S_4 . Here the parentheses represent the cycle structures. Find out the group element g of S_4 that conjugates g_1 and g_2 as, [3]

$$g^{-1} \circ g_1 \circ g = g_2.$$

2. Consider a group G of order N .

- a. Prove that the identity element e in G is unique. [2]
- b. Prove that an element g of G has a unique inverse. [2]

- c. Show that the multiplication table for order $N = 3$ group is unique. Write it down. [3+1]
3. a. Show that \mathbb{Z}_3 is a subgroup of S_3 . [3]
 b. Show that the order of a subgroup is a factor of the group order. [3]
 c. Find out the **factor group** S_3/\mathbb{Z}_3 . [2]
4. Consider the **defining representation** of the permutation group S_n .
 a. Show that each group element has a single 1 in each row and column, rest are zero. [2]
 b. Show that this representation is reducible. [3]
 c. Show that if this group was Abelian, then any irreducible representation of it would have been one-dimensional. [3]
5. What is Schur's Lemma (Don't go on proving it!)? What does it mean for a quantum mechanical wave function and the corresponding physical observable if the system has some symmetry? [2+2+3]
6. a. Argue (Don't prove!) why the number of conjugacy classes is equal to the number of irreducible representations of a group. [3]
 b. The permutation group S_3 has elements $\{e, \alpha, \beta, C, D, E\}$. What are the conjugacy classes of this group and how do they differ qualitatively? [1+2]
 c. Construct the character table for the group S_3 ,

	C_x	C_y	\dots
D_0	?	?	\dots
D_1	?	?	\dots
D_2	?	?	\dots

wherein $C_{x,y,\dots}$ stand for different conjugacy classes and $D_{0,1,2}$ are the three irreducible representations. Among them, D_0 is the trivial representation. You may use the two orthogonality conditions among characters:

$$\frac{1}{N} \sum_x k_x \chi_a^*(g_x) \chi_b(g_x) = \delta_{ab},$$

$$\frac{k_x}{N} \sum_a \chi_a^*(g_x) \chi_a(g_y) = \delta_{xy},$$

where N is the order of the group, k_x is the order of the conjugacy class C_x and $\chi_a(g_x)$ stands for the character for conjugacy class C_x in representation D_a . [10]

Best wishes