



**Assignment 3**  
Ph. D. Coursework, NAS-MUNA  
**Symmetries & Lie Algebra in Physics**  
(NWTP 702)  
Instructor: Kumar Abhinav  
Date: October 30, 2023

Due on November 9, 2023

Semester 1

Total marks: 24

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**Instructions**

- I. Submit answers both in hard and soft (through e-mail) copies. Do not waste time by typing it out.
- II. Use either blue or black ink.
- III. Delay in submission may reduce marks.
- IV. Individual marks are given in parentheses.
- V. Attempt any 3 of the 4 questions given.

**Questions**

1. a) Show that in case of a Lie group the group multiplication amounts to,

$$e^{i\alpha_a X_a} e^{i\beta_b X_b} = e^{i\gamma_c X_c},$$

where  $\alpha_a, \beta_a, \gamma_a$  are continuous parameters and the Lie generators satisfy,

$$[X_a, X_b] = i f_{abc} X_c,$$

with  $f_{abc}$  being the structure constants. [6]

- b) Verify the Jacobi identity for a Lie algebra. [2]
2. a) Find out the  $SU(2)$  generators in the matrix form in their adjoint representation. [2+2+2]
- b) Why do they represent a  $4\pi$  rotational symmetry? [2]

3. Derive  $SO(3)$  generators as differential operators. [2+2+2+2]

4. Consider the  $SU(2)$  generators,

$$J^{\pm} = \frac{1}{\sqrt{2}} (J_1 \pm iJ_2) \quad \text{and} \quad J_3.$$

In the eigenbasis of  $J_3$ ,

$$J_3|m\rangle = m|m\rangle,$$

find out the coefficients  $N_m$  such that,

$$J^-|m\rangle = N_m|m\rangle.$$

[8]

**Best wishes**