



Assignment 4
Ph. D. Coursework, NAS-MUNA
Symmetries & Lie Algebra in Physics
(NWTP 702)
Instructor: Kumar Abhinav
Date: November 10, 2023

Due on November 19, 2023

Semester 1

Total marks: 30

Instructions

- I. Submit answers both in hard and soft (through e-mail) copies. Do not waste time by typing it out.
- II. Use either blue or black ink.
- III. Delay in submission may reduce marks.
- IV. Individual marks are given in parentheses.

Questions

1.
 - a) Write down the Poincaré generators and compute their commutators. [5]
 - b) Show that the Lorentz algebra can be factorized as two $so(3)$ algebras. [5]
2.
 - a) What are the weights and roots of a Lie algebra? Show that in the adjoint representation, the states corresponding to the Cartan generators have zero weight. [2+3]
 - b) The proton and neutron states are defined as,

$$a_p^\dagger|0\rangle = |p\rangle, \quad \text{and} \quad a_n^\dagger|0\rangle = |n\rangle,$$

respectively, where $|0\rangle$ stands for the nucleon vacuum. Calculate the eigenvalue of the number operator $N = a_p^\dagger a_p + a_n^\dagger a_n$ for the state, [5]

$$a_{p_1}^\dagger \cdots a_{p_k}^\dagger a_{n_1}^\dagger \cdots a_{n_l}^\dagger |0\rangle = |p_1, \cdots, p_k, n_1, \cdots, n_l\rangle.$$

3. 1. Draw the different types of Young Tableaux possible for $SU(3)$. [3]
2. Construct a covariant derivative D_μ such that the following covariance condition is held: [4]

$$D_\mu (e^{i\theta(x)} \phi(x)) = e^{i\theta(x)} D_\mu \phi(x).$$

3. Show that the $SU(N)$ gauge field tensor,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i [A_\mu, A_\nu],$$

transforms as $UF_{\mu\nu}U^{-1}$ under some representation U given that the gauge field itself transforms as, [3]

$$A_\mu \rightarrow UA_\mu U^{-1} + i (\partial_\mu U) U^{-1}.$$

Best wishes