Centre for Theoretical Physics & Natural Philosophy
"Nakhonsawan Studiorum for Advanced Studies" - NAS

Final Examination

Ph. D. Coursework, NAS-MUNA

Symmetries & Lie Algebra in Physics

(NWTP 702)

Instructor: Kumar Abhinav

Date: November 23, 2023

Duration: 12:00 to 15:00 hrs | **Open Book Format** | Total marks: 30

Instructions

I. All kinds of notes and books are allowed. However, the use of the Internet (not computer!) is prohibited.

III. Discussion among students is strictly prohibited.

III. Attempt **any 3** of the 7 questions given. In any case only the first 3 questions attempted will be marked.

IV. Use either blue or black ink.

V. Individual marks are given in parentheses.

Questions

1. a) Show that every representation of a *finite* group is equivalent to the unitary representation. Why this is not true for infinite (not continuous!) groups. [5+1]

   b) What are the characters of a group? Show that a conjugacy class corresponds to a unique character. [1+3]

2. Consider the permutation group of 3 objects (a, b, c).

   a) Construct all the Young tableaux and associated states. [6]

   b) Show that the sub-space associated with the mixed tableaux corresponds to a two-dimensional irreducible representations. [4]

3. a) Find out the expression for,

$$\frac{\partial}{\partial \alpha_a} \exp\left(\frac{i}{2}\alpha_b\sigma_b\right),$$

for the $su(2)$ algebra. Here σ_a $(a = 1, 2, 3)$ are the Pauli matrices and α_a are continuous parameters. [8]

b) Show that, [2]

$$\frac{\partial}{\partial \alpha_a} \exp\left(\frac{i}{2}\alpha_b\sigma_b\right)\Bigg|_{\alpha_c=0\forall c=1,2,3} = \frac{i}{2}\sigma_a.$$

4. Construct all the sub-spaces spanned by the irreducible representations generated through combining two $SU(2)$ particles with highest weights $j_1 = 1$ and and $j_2 = 1/2$. [10]

5. a) What is the $O(3)$ group? From the conservation of chirality, show that $\det R = \pm 1$ where $R \in O(3)$. [1+3]

b) Construct the $O(3)$ generators T_a $(a = 1, 2, 3)$ such that,

$$R = \exp\left(i\omega_a T_a\right),$$

where ω_a are rotation angles about the respective axes. [6]

6. a) In the adjoint representation, show that the states can be labelled as generators. [2]

b) Show that the state $|H_i\rangle$ corresponding to the Cartan generators H_i have zero weight. [3]

c) Show that the non-Cartan generators E_α behave as raising/lowering operators to the eigenstates of H_i. [5]

7. Derive the super-selection rule for the two-nucleon state. [10]

Best wishes