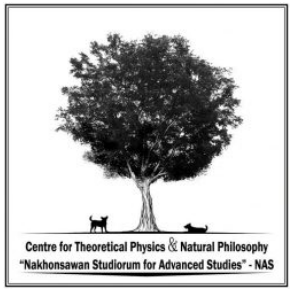

Assignment 1

Ph. D. Coursework, NAS-MUNA
Symmetries & Lie Algebra in Physics
(NWTP 702)
Instructor: Kumar Abhinav
Date: February 15, 2025

Due on March 1, 2025 | Semester 2/2024 | Total marks: 35

Instructions

I. All questions are mandatory.

II. Submit answers both in hard and soft (scanned) copies. Do not waste time by typing it out.

III. Use either blue or black ink.

IV. Delay in submission may reduce marks.

V. Individual marks are given in parentheses.

1. Consider a **rectangle** (a, b, c, d) in a **2-dimensional space** (If you consider this space to be the paper you are writing in, then the rectangle cannot move out of the paper).
   i) Identify the symmetry group of this system. [2]
   ii) Construct the multiplication table of that group. [2]
   iii) Construct a **non-trivial representation** for this group and identify whether it is reducible or irreducible. [3+1=4]
   iv) Find out the subgroup(s) of this group and construct the corresponding coset(s). [2+2=4]
   ii) Find out the conjugacy classes of this group. [3]

2. You know the **conjugacy classes** and **irreducible representations** of the group S_3 (No need to write them down). Construct the **character table** (not the multiplication table) for this group from this information. [5]

3. From the orthogonality relation,
$$\sum_{g\in G}\left[D^{(a)}(g)\right]_{jk}\left[D^{(b)}(g)\right]_{lm}=\frac{N}{n_a}\delta_{ab}\delta_{jl}\delta_{km},$$
of **matrix elements** of a group representations $D^{(a,b)}(g)$ of **group order** N and representation dimension n_as argue that,
$$N=\sum_a n_a^2.$$
There is **no need** to derive the orthogonality relation itself. [5]

4. Consider a spin-1 object. The complete states are given as $|m,\, x\rangle$, where $m = -1, 0, 1$ is the magnetic quantum number and x represents all other variables. The orthogonality relation is,
$$\langle m,\, x|m',\, y\rangle = \delta_{mm'}\delta_{xy}.$$
   i) Construct the **matrices** that connects these states with different m but same x. [5]
   ii) **Identify** the group and construct the multiplication table. [1+4=5]

Best wishes