Centre for Theoretical Physics & Natural Philosophy
"Nakhonsawan Studiorum for Advanced Studies" - NAS

Assignment 2

Ph. D. Coursework, NAS-MUNA

Integrable System

(NWTP 601)

Instructor: Kumar Abhinav

Date: October 28th, 2025

Due on November 13th, 2025 | Semester 1/2025 | Total marks: 30

Instructions

I. Attempt all questions.

II. Do not waste time by typing the answers.

III. **DO NOT** use red ink.

IV. No extension of the deadline will be considered.

V. Individual marks are given in parentheses.

Questions:

1. a) Show that the Poisson bracket in covariant notations,

$$\{y^\mu,\, y^\nu\}\,,$$

where y^μ are phase-space variables, transform like a second rank contravariant tensor under a canonical transformation $y^\mu \to z^\mu(y)$. [5]

b) Given the Poisson bracket satisfy the Jacobi identity, show that its **inverse** satisfy the Bianchi identity (Perform all the steps of the derivation clearly!). [7]

2. a) Show that although the KdV system is Galilean invariant, the mKdV system is not. [5]

b) Considering the recursion relation for KdV/mKdV conserved charge densities (modulo a factor of $3(-1)^n$ for v_n),

$$v_n + i\partial_x v_{n-1} + \frac{1}{6}\sum_{m=0}^{n-2} v_{n-m-2}v_m = 0,$$

and the fact that $v_0 = u$, where u is the KdV solution, obtain the next 4 conserved charge densities ofthis system. [1+1.5+2.5+3]

c) Given the recursion relation,

$$\left(\partial_x^3 + \frac{1}{3}\left(\partial_x u(x) + u(x)\partial_x\right)\right)\frac{\delta H_{n-1}[u]}{\delta u(x)} = \partial_x \frac{\delta H_n[u]}{\delta u(x)},$$

among the conserved charges $\{H_n[u]\}$ of the KdV/mKdV systems, show that they are in involution:

$$\{H_n[u],\, H_m[u]\}_1 = 0,$$

wherein $\{,\}_1$ stands for the KdV Poisson bracket of the first kind: [5]

$$\{u(x),\, u(y)\} = \partial_x \delta(x-y).$$

Best wishes