Centre for Theoretical Physics & Natural Philosophy
"Nakhonsawan Studiorum for Advanced Studies" - NAS

Assignment 1

Ph. D. Coursework, NAS-MUNA

Mathematical Methods of Physics

(NWTP 501)

Instructor: Kumar Abhinav

Date: March 7th, 2026

Due on March 21st, 2026 | Semester 2/2025 | Total marks: 41

Instructions

I. Attempt all questions.

II. Submit answers both in hard and soft (scanned) copies. Do not waste time by typing it out.

III. Use either blue or black ink.

IV. Delay in submission may reduce marks.

V. Individual marks are given in parentheses.

1. Show that additive identity is unique whereas for each element of a field there is a unique inverse under addition. [2+2]

2. Show that the set of all real numbers of the form $a + \sqrt{2}b$, with a and b being *rational numbers*, is a field. But if a and b are integers then show that the same set is a ring only. [3+2]

3. a) Consider a vector space homomorphism:

$$T : V \to W.$$

Show that under T the identity of V is uniquely mapped to that of W. [2]

b) Show that $\mathbb{R}^\infty$ is isomorphic to a proper subspace of itself. [5]

c) Show that $\{(x, y)|x = y\}$ is a proper subspace of $\mathbb{R}^2$ whereas $\{(x, y)|x = y + 1\}$ is not. [2+2]

4. a) Consider the basis expansion:

$$v = v^i e_i,$$

of a vector. Show that if the basis vector e_i transforms covariantly then the component v^i transforms contravariantly under a linear transformation. [3]

b) Consider the metric tensor,

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix},$$

corresponding to a generic basis $\{u_i\}$. Obtain an orthogonal basis $\{e_i\}$ for this vector space and determine its signature. [4+1]

c) Show that the signature of a vector space is basis-independent. [5]

5. Show that,

a) Hermiticity of an operator remains invariant over a basis transformation. [2]

b) A unitary transformation preserves the inner product. [2]

c) A hermitian operator has real eigenvalues and orthogonal eigenstates. [2+2]

Best wishes