Centre for Theoretical Physics & Natural Philosophy
"Nakhonsawan Studiorum for Advanced Studies" - NAS

End Semester Examination

Ph. D. Coursework, NAS-MUNA

Mathematical Methods of Physics

(NWTP 501)

Instructor: Kumar Abhinav

Date: May 22nd, 2026

Time 10:00 - 13:00 hrs

Total time: 180 minutes | Semester 2/2025 | Total marks: 30

Instructions

I. Attempt **ALL** questions.

II. **ONLY** your class notebook(s) are allowed. **NO** phones and the Internet are allowed.

III. Use blue or black ink **ONLY**.

IV. Individual marks are given in parentheses.

V. Read the questions carefully. Try to write precisely what is asked for; you will **NOT** get extra marks for writing unnecessarily long answers. For example, a 1 mark answer should not be more than 2-3 lines.

Questions:

1. Explain, in your own words, why cannot we have an analog of the Cauchy-Riemann condition in two-dimensional real space. [2]

2. Consider the complex function,
$$\omega(z) = (z-2)(z+3)^{1/3}.$$
   a) Will it be multi-valued? If yes then for what value of z? [1+1]
   b) If yes, determine the branch point(s) and its(their) order. [1+1]
   c) Considering the interval $[-\pi, \pi]$ of the angular variable, obtain the branch-cut (if any) by drawing it out (Just the diagram, you don't need to write anything). [2]

3. Solve the following integral,
$$I = \int_0^{2\pi} \frac{d\theta}{2+\sin(\theta)}.$$
   Hint: $2 > \sqrt{3} > 1$. [5]

4. a) Show that, for any $m, n \in \mathbb{Z}$, $\sin(mx)$ and $\cos(nx)$ are mutually orthogonal in any even domain of integration. [2]
   b) Find the Fourier series of $f(x) = x$ in $[-\pi, \pi]$. [5]

5. a) Consider the Frobenius series,
$$y(x) = \sum_{n=0}^{\infty} a_n x^{n+s}.$$
   Find out the possible values of s for the equation, [2]
$$x^2 y'' + (2+x)y = 0.$$

b) Consider the operator Dx^n where $D = d/dx$ and $n \in \mathbb{Z}_+$. Show that,

$$[D,\, x^n] = nx^{n-1},$$

where [,] is the commutator bracket. [2]

c) The generating function for the Legendre polynomials is,

$$\Phi(x,h) = \sum_{n=0}^{\infty} h^n P_n(x) = \frac{1}{\sqrt{1-2xh+h^2}}.$$

Using i, derive the following recursion relation,

$$P_l'(x) - 2xP_{l-1}'(x) + P_{l-2}'(x) = P_{l-1}(x),$$

for the Legendre polynomial $P_n(x)$. [3]

d) Evaluate the integral, [3]

$$I = \int_0^{\infty} e^{-x^2}\, dx.$$

Best wishes