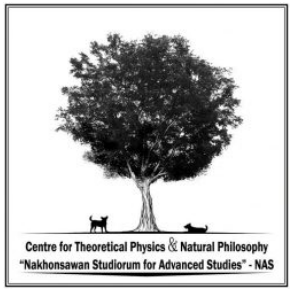

Mid-Semester Examination

Ph. D. Coursework, NAS-MUNA

Mathematical Methods of Physics

(NWTP 501)

Instructor: Kumar Abhinav

Date: March 23rd, 2025

Time 09:00 - 11:00 hrs

Total time: 120 minutes | Semester 2/2025 | Total marks: 20

Instructions

I. Attempt **any 4** out of the 6 questions given.

II. **ONLY** your class notebook(s) are allowed.

III. Use blue or black ink **ONLY**.

IV. Try to submit on time.

V. Individual marks are given in parentheses.

Questions:

1. Consider the set of vectors:

$$v_1 = x + y + z, \quad v_2 = x - y, \quad v_3 = y - z,$$

in a 3-dimensional real vector space, where (x, y, z) represents an *orthonormal basis*. Construct another orthonormal basis from the set (v_1, v_2, v_3) through the Gram-Schmidt procedure. [5]

2. Show that, for any symmetric matrix S to remain symmetric under a similarity transformation:

$$S \to S' = T^{-1}ST,$$

the transformation matrix T needs to be orthogonal. [5]

3. a) For the set $X = \{a, b, c\}$ construct one nontrivial topology containing elements in addition to $\{\phi\}$ and X. [3]

   b) What is the ϵ-neighborhood of a point in a vector space? Why it cannot be defined in a non-metric space? [1+1]

4. a) Show that the metric operator:

$$H = [h_{ij}], \quad h_{ij} = \langle e_i | e_j \rangle,$$

corresponding to a generic basis $\{e_i\}$ in a *complex* vector space is Hermitian. [3]

   b) Show that H defined above remains Hermitian under a basis transformation $e'_j = A^i_j e_i$. [1+1]

5. a) Plot $z = \ln(-5)$ in the complex plane (You don't need to be accurate; just mark the coordinates corresponding to z on the real and imaginary axis). [2]

b) Solve for Z: [3]

$$Z^3 + 2 = 0.$$

6. a) What a unit circle in the z-plane will map to in the w-plane under the map, [2]

$$w = \frac{z-1}{z+1}?$$

b) Check whether $w = \ln z$ is analytic or not (Hint: $\frac{d}{dx}\arctan(x) = \left(1+x^2\right)^{-1}$). [3]

Best wishes